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On the anomalous dimension of the transversity distribution $h_1(x,Q^2)^\star$

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Abstract. We calculate the leading order anomalous dimension of the transversity structure function directly using three different methods, the local light–cone expansion in the forward case, the non–forward case, and the short–distance expansion of the forward Compton amplitude. Our results agree with the original calculation by Artru and Mekhfi [Z. Phys. **45** (1990) 669], which has been doubted recently. We also comment on the next-to-leading order anomalous dimension.

1 Introduction

The transversity distribution $h_1(x,Q^2)$ [1,2] is one of the central inclusive quantities which emerge in the transverse polarized Drell–Yan process and deep inelastic scattering off targets with a spin $J \geq 1$. This function plays also a crucial rule in proving sum rules for other inclusive functions in deep inelastic scattering off polarized targets [3]. The transversity distribution is a flavor non–singlet function and receives contributions starting with twist–2. The experimental measurement of $h_1(x,Q^2)$ is currently planned at different facilities [4] and the scaling violations of this quantity may ultimately be compared with the predictions of perturbative Quantum Chromodynamics and thus serve as a novel test of this theory.

The leading order (LO) splitting function $P_{h_1}(z)$ for the distribution $h_1(x,Q^2)$ has first been calculated by Artru and Mekhfi [5] using time-ordered or 'old-fashioned' perturbation theory [6] for the range of momentum fractions $z < 1$ determining the end–point contribution as in the case of the longitudinally unpolarized or polarized flavor non–singlet distribution $P_{NS}(z)$ [7], where the conservation of fermion–number allows this. For the transversity distribution such an integral relation does not exist a priori. Therefore one may doubt this procedure, as has been done in [8] recently, where a different result was obtained for the splitting function $P_{h_1}(z)$ in a direct calculation using a method by Ioffe and Khodjamirian [9]. As a result the authors of [8] conclude that either the splitting function $P_{h_1}(z)$ was wrongly calculated in the past or the method of [9] is inapplicable.

It is the aim of the present paper to clarify this problem, which is of importance for the correct understanding of the evolution of the transversity distribution $h_1(x,Q^2)$. We will calculate the anomalous dimension of the distribution $h_1(x,Q^2)$ by three different methods. Moreover, it is interesting to know, whether one can determine anomalous dimensions using the method [9], which may be of special importance for time–like higher order calculations, i.e. in a situation where the light–cone expansion is *inapplicable*. A related question as raised in [8] on the correct end–point behaviour indeed occurs also in next-to-leading order, since the splitting function there also was evaluated under some assumptions [10], due to the techniques being applied, see [11,12].

The paper is organized as follows. In Sect. 2 we recalculate the forward anomalous dimension of $h_1(x,Q^2)$ in leading order using the local light–cone expansion, by calculating the non–forward anomalous dimension, from which the forward limit is derived, and the short–distance expansion of a related Compton amplitude [9]. In Sect. 3 we comment on the next-to-leading order anomalous dimension of $h_1(x,Q^2)$ and Sect. 4 contains the conclusions.

2 LO anomalous dimension of $h_1(x,Q^2)$

2.1 Forward-scattering anomalous dimension

We first calculate the anomalous dimension using the local light cone expansion in the forward case, $p_1 = p_2 = p$. The contributing diagrams are depicted in Fig. 1. The Feynman rules for the vector–valued operator insertions for two quark and two quark and a gluon line read

$$O_n^{(0)\mu}.\Delta = \sigma^{\mu\nu}\Delta_\nu (p.\Delta)^n \tag{1}$$

$$O_n^{(1)\mu\lambda}.\Delta = g t^a \sigma^{\mu\nu}\Delta_\nu \sum_{j=1}^{n} (p'.\Delta)^{j-1}\Delta^\lambda (p.\Delta)^{n-j} \ . \tag{2}$$

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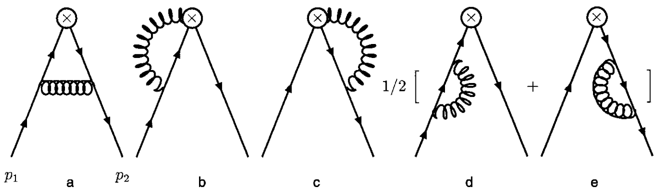

Fig. 1a–e. The diagrams for the LO anomalous dimension for (non) forward scattering. ⊗ are the operator insertions (1,2). The curled and straight lines denote the gluon and quark lines, respectively

They are denoted by the symbol ⊗ in Fig. 1. Here, p' is the second quark momentum at the quark-quark-gluon insertion Fig. 1b,c, $\sigma_{\mu\nu} = (i/2)[\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu]$, Δ_ρ is a light–like vector with $\Delta.\Delta = 0$, t^a denotes the $SU(3)_c$ generators, g is the strong coupling constant, and λ the vector index of the additional gluon.

The anomalous dimension $\gamma_n(g(\mu))$ is obtained by

$$\gamma_n(g(\mu)) = \mu^2 \frac{\partial}{\partial \mu^2} \ \ln Z_n(\mu) \ , \tag{3}$$

with

$$O_n(\mu) = Z_n^{-1}(\mu) \ O_n^{\rm bare} \ . \tag{4}$$

In leading order the anomalous dimension is a gauge invariant quantity[1]. We will work, for definiteness, in the Feynman–gauge. The contributions to the Z–factor in the forward case will be denoted by I_j with $j = a, ..., e$, cf. Fig. 1.

In the limit $D = 4 - \varepsilon \to 4$ the term I_a vanishes in the Feynman–gauge since the contraction of the Dirac–matrices in the numerator results into a term $\propto \varepsilon$, which cancels the pole of the loop–integral. The Z-factor, keeping the quark momentum off-shell, is

$$Z_n(\mu) = 1 + 2a_s C_F \left[\frac{2}{\varepsilon} - \ln\left(\frac{-p^2}{\mu^2}\right)\right] \left\{1 + 4\sum_{j=2}^{n} \frac{1}{j}\right\} \ , \tag{5}$$

with $a_s = \alpha_s/(4\pi)$, $\alpha_s = g^2/(4\pi)$, and $C_F = 4/3$. The first and second summand in the brackets are due to $I_b + I_c$ and $I_d + I_e$, respectively. By (3) the anomalous dimension

$$\begin{aligned}\gamma_n^{h_1}(g) &= a_s C_F \left\{1 + 4\sum_{j=2}^{n} \frac{1}{j}\right\} \\ &= 2a_s C_F \left\{-\frac{3}{2} + 2S_1(n)\right\} \equiv -\int_0^1 dz z^{n-1} P_{h_1}(z)\end{aligned} \tag{6}$$

is obtained in agreement with the result of [5], where $S_1(n) = \sum_{j=1}^{n}(1/j)$. Note that we did not assume any sum–rule in this derivation. The splitting function $P_{h_1}(z)$ reads

$$P_{h_1}(z) = 2a_s C_F \left\{-2 + \frac{2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right\} \ . \tag{7}$$

[1] In the $\overline{\rm MS}$ scheme this holds to all orders

2.2 Non-forward-scattering anomalous dimension

The diagrams in Fig. 1 can also be used to calculate the anomalous dimension of $h_1(x, Q^2)$ in the non–forward case, $p_1 \neq p_2$. The operator insertions corresponding to (1,2) read:

$$O^\mu = -i\tilde{x}_\nu \sigma^{\mu\nu} e^{i\tilde{x}.p_+\kappa_+} \left[e^{i\tilde{x}.p_-\kappa_-} - e^{-i\tilde{x}.p_-\kappa_-}\right] \tag{8}$$

$$\begin{aligned}O^{\mu\lambda} &= igt_a \tilde{x}_\nu \sigma^{\mu\nu} \tilde{x}^\lambda e^{i\tilde{x}.p_+\kappa_+} \\ &\times \left[e^{i\tilde{x}.p_1\kappa_-} - e^{-i\tilde{x}.p_1\kappa_-}\right] \frac{e^{i\tilde{x}.k\kappa_2} - e^{i\tilde{x}.k\kappa_1}}{\tilde{x}.k} \ .\end{aligned} \tag{9}$$

Here we follow the notation of [13], where the corresponding insertions for the unpolarized scalar operators were given. $\tilde{x}$ is a light–like vector, λ the gluon vector index and k the gluon momentum. κ_1 and κ_2 are light-cone marks with $\kappa_\pm = (\kappa_2 \pm \kappa_1)/2$, and $p_\pm = p_2 \pm p_1$. We work in the Feynman–gauge again and choose $\kappa_+ = 0, \kappa_- = 1$ in the following.

There exist various equivalent representations for the non–forward splitting functions, which are known as $\alpha-$, $w-$, and near–forward representation, see [13–16]. We will first refer to the α–representation, which is directly related to the Feynman–parameter representation of the diagrams in Fig. 1. Let us denote the different contributions to the non–forward $Z-$factor, cf. [15], by J_k, $k = a, ..., e$. J_a vanishes for the same reason as in the forward case. In the α–representation the non–forward splitting function is obtained by

$$\begin{aligned}K^{h_1}(\alpha_1, \alpha_2) = \frac{\alpha_s}{2\pi} C_F &\left\{\left[-\delta(\alpha_1) - \delta(\alpha_2) + \delta(\alpha_1)\left(\frac{1}{\alpha_2}\right)_+\right.\right. \\ &\left.\left. +\delta(\alpha_2)\left(\frac{1}{\alpha_1}\right)_+\right] + \frac{3}{2}\delta(\alpha_1)\delta(\alpha_2)\right\} \ .\end{aligned} \tag{10}$$

The $[]_+$ distribution is defined by

$$\int_0^1 dx[f(x,y)]_+ \phi(x) = \int_0^1 f(x,y)[\phi(x) - \phi(y)], \tag{11}$$

where f is a distribution out of the space $\mathcal{D}'[0,1] \times [0,1]$ and $\phi(x) \ \epsilon \ \mathcal{D}[0,1]$ a general basic function, [17]. The first addend in (10) is due to $J_b + J_c$ and the second due to $J_d + J_e$. Equation (10) agrees with a result in [18], see also [19]. There are different possibilities to derive the forward anomalous dimension from (10). One can either use the near–forward representation and perform the limit $\tau = \tilde{x}.p_-/\tilde{x}.p_+ \to 0$, cf. Appendix D in [15], or perform a direct integral in the $\alpha-$ or $w-$representation [14,15], or covert K^{h_1} into the local representation with respect of the two Mellin–variables n and n', cf. [13,15] and obtain the forward anomalous dimension by setting $n = n'$.

We first change to the $w-$representation

$$K^{h_1}(\alpha_1, \alpha_2) D\alpha = \widetilde{K}^{h_1}(w_1, w_2) Dw, \tag{12}$$

with

$$w_1 = \alpha_1 - \alpha_2, \quad w_2 = 1 - \alpha_1 - \alpha_2 \ , \tag{13}$$

$$\widetilde{K}^{h_1}(w_1, w_2)$$
$$= -2a_sC_F\Bigg\{\delta(1-w_2+w_1)\left[1-\frac{2}{(1-w_2-w_1)_+}\right]$$
$$+\delta(1-w_2-w_1)\left[1-\frac{2}{(1-w_2+w_1)_+}\right]$$
$$-\frac{3}{2}\delta(w_1)\delta(1-w_2)\Bigg\}\,, \qquad (14)$$

and

$$D\alpha = d\alpha_1 d\alpha_2\left[\theta(\alpha_1)\theta(\alpha_2)\theta(1-\alpha_1-\alpha_2)\right.$$
$$\left.+\theta(1-\alpha_1)\theta(1-\alpha_2)\theta(\alpha_1+\alpha_2-1)\right] \qquad (15)$$
$$Dw = \frac{1}{2}dw_1 dw_2\left[\theta(1+w_1-w_2)\theta(1-w_1-w_2)\theta(w_2)\right.$$
$$\left.+\theta(1+w_1+w_2)\theta(1-w_1+w_2)\theta(-w_2)\right] \qquad (16)$$

The forward scattering splitting function is derived by

$$P^{h_1}(z) = \int_{-1+z}^{1-z} dw_1 \widetilde{K}^{h_1}(w_1, z), \qquad (17)$$

cf [14,15]. One obtains from (14,17)

$$P_{h_1}(z) = 2a_sC_F\left\{-2+\frac{2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right\}\,. \qquad (18)$$

in agreement with [5].

Likewise one may first calculate the local non–forward anomalous dimension $\gamma_{h_1}^{nn'}$

$$\gamma_{h_1}^{nn'} = 2a_sC_F\left\{\left[-\frac{3}{2}+2S_1(n)\right]\delta_{nn'}-\frac{2}{n-n'}\frac{n'}{n}\theta_{nn'}\right\}, \qquad (19)$$

where $\delta_{nn'}$ denotes the Kronecker symbol and $\theta_{nn'} = 1$ for $n' < n$ and 0 otherwise. In the forward limit $n = n'$ (6) is obtained in agreement with [5].

2.3 Derivation of the anomalous dimension from a forward compton amplitude

The anomalous dimension for $h_1(x, Q^2)$ may be also calculated form the short distance expansion of the Compton amplitude of an axial-vector and scalar current j_1 and j_2 directly using the method by Ioffe and Khodjamirian [9]. This has been tried in [8] recently. To extract the anomalous dimension of the corresponding operator matrix element in this method we start to write down the renormalization group equation (RGE) for the Green's function $F(q; p_1, \ldots, p_n)$, the Fourier transform of $\langle 0|T[j_1(x)j_2(0)\,\phi(x_1)\ldots\phi(x_n)]|0\rangle$,

$$\left[\mathcal{D}+\gamma_{j_1}(g)+\gamma_{j_2}(g)-n\gamma(g)\right]F(q; p_1, \ldots, p_n) = 0\,, \qquad (20)$$

with $n = 2$. The RG-operator is given by

$$\mathcal{D} \equiv \mu^2\frac{\partial}{\partial\mu^2}+\beta(g)\frac{\partial}{\partial g}-\gamma_m(g)m\frac{\partial}{\partial m}\,, \qquad (21)$$

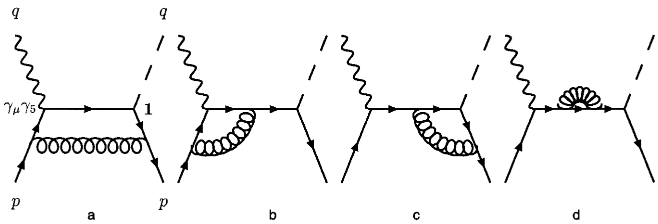

Fig. 2a–d. QCD 1–loop diagrams to the forward Compton amplitude

and $\gamma_{j_k}(g), \gamma(g), \gamma_m(g)$ are the anomalous dimensions of the currents j_k, the outer legs, and the mass, respectively. $\beta(g)$ denotes the $\beta-$function and μ is the factorization scale. At short distances the Green's function has the representation

$$F(q; p_1, \ldots, p_n) = \sum_k C_k(q)E^k(p_1, \ldots, p_n)\,, \qquad (22)$$

with $C_k(q)$ the Wilson coefficients and $E^k(p_1, \ldots, p_n)$ the hadronic matrix elements. Evaluating the Compton amplitude Fig. 2 one obtains logarithmically divergent contributions which belong to the RGE of the Wilson coefficient,

$$\left[\mathcal{D}+\gamma_{j_1}(g)+\gamma_{j_2}(g)-\gamma_{O_k}(g)\right]C_k(q) = 0\,. \qquad (23)$$

The corresponding RGE for the hadronic matrix elements E^k follows from (20,23). Here $\gamma_{O_k}(g)$ is the anomalous dimension of the composite operator $O_k(0)$ being obtained from

$$j_1(x)j_2(0) = \sum_k C_k(x)O_k(0)\,. \qquad (24)$$

$\gamma_{O_k}(g)$ is therefore given by (3,5,6). The term $\propto \ln(-p^2/\mu^2)$ which results from the diagrams of Fig. 2 is

$$\gamma_C(g) = \gamma_{j_1}(g)+\gamma_{j_2}(g)-\gamma_{O_k}(g)$$
$$= -2a_sC_F\cdot 2S_1(n)\,. \qquad (25)$$

The anomalous dimensions of the two currents are [20]

$$\gamma_{j(s)} = -C_Fa_s(s-1)\Big\{(s-3)+\frac{a_s}{18}\Big[4(s-15)T_FN_f$$
$$+(18s^3-126s^2+163s+291)C_A$$
$$-9(s-3)(5s^2-20s+1)C_F\Big]\Big\}+O(a_s^3)\,, \qquad (26)$$

where s denotes the spin, i.e. $\gamma_{j(s=1)} \equiv 0$ and

$$\gamma_{j(s=0)} = -3C_Fa_s+C_F\left[\frac{10}{3}T_FN_f-\frac{97}{6}C_A-\frac{3}{2}C_F\right]a_s^2$$
$$+O(a_s^3)\,, \qquad (27)$$

which yields the splitting function, (9). This shows that the method of [9] can be used to calculate the anomalous dimensions.

3 A remark on the anomalous dimension of $h_1(x,Q^2)$ at NLO

The twist–2 NLO anomalous dimension and splitting function for the distribution $h_1(x,Q^2)$ was calculated in [10] using the cut vertex method of [11] or the method outlined in [12]. In both cases the $\delta(1-z)$–contribution cannot be fixed easily by a direct calculation, see also [21], since the calculation is performed in a physical gauge. In fact, a complete calculation of these terms is not yet available. On the other hand, complete calculations of the NLO anomalous dimensions as occurring for the various polarized and unpolarized non–singlet and singlet twist–2 parton densities contributing to the structure functions exist in the R_ξ–gauges, cf. [22]. Thus these direct calculations assure that for the non–singlet '$-$' twist–2 quark splitting functions fermion number conservation holds, as energy–momentum conservation holds in the singlet sector.

Knowing this result and working in a class of gauges in which the NLO anomalous dimensions are gauge invariant one may take advantage in comparing the $\delta(1-z)$–terms occurring in the quark splitting functions $P^q_{NS^-}(z)$ and $P_{h_1}(z)$. This is now done in the axial gauge. One classifies the contributing Schwarz *distributions* dealing with the $+$–distribution as an individual one and *not* being expressed due to a composed 'function' of $\delta-$ and Heaviside functions in the limit $\varepsilon \to 0^+$. Then all the $\delta(1-z)$ terms arise from the self-energy terms, which are *independent* of the operator insertion. Due to this they can be determined without an explicit calculation, completing the calculation of the splitting function. The result on the forward scattering splitting functions has been extended to the case of non–forward scattering in [23].

4 Summary

We have shown by three different complete calculations that the calculation of the leading order anomalous dimension of the transversity distribution $h_1(x,Q^2)$ as originally being derived in [5] is correct. Our results also agree with those which have been obtained in [24] very recently. It was shown that the method of Ioffe and Khodjamirian can be used to derive anomalous dimensions in the present example and similarly for related cases if applied to coefficient functions $C_k(x)$, contrary to the result of [8], which has to be regarded to be wrong. The methor of [24] may thus be of importance for higher order calculations in situations in which the light cone expansion does not apply. Furthermore we argued that the $\delta(1-z)$ terms of the next-to-leading order calculations, although not yet being calculated directly, are right.

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Note added in proof: After this paper was finished D. Boer pointed out to me that the anomalous dimension of $h_1(x, Q^2)$ was calculated prior to Artru and Mekhfi [5] by Shifman and Vysotsky [25] and at the same time by Baldraccini et al. [26]. Later also Bukhvostov et al. calculated a Matrix element in the quasipartonic approximation which can be interpreted as the LO anomalous dimension [27]. I would like to thank R. Kirschner for this remark. We also note that the anomalous dimension of $h_1(x, Q^2)$ at small x was calculated in [28].